**Assignment 3**

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Part 1

1. A) Calculate the output generated by the network in response to this input. Show the process of output generation, including the z and 𝒂 values for each neuron in the network.

Solution:

For Neuron 3 (first hidden neuron):

z3 = (−0.1⋅0.3)+(−0.2⋅0.6) + 0.1 \* 1

= −0.03 −0.12 + 0.1

z3 = −0.05

For Neuron 4 (second hidden neuron):

z4 = (0.2⋅0.3)+(0.3⋅0.6)+0.1

= 0.06 + 0.18 + 0.1

Z4 = 0.34

On applying the rectifier linear activation function to z3​ and z4​:

a3 = max (0, −0.05) = 0

a4 = max (0, 0.16) = 0.34

Calculate z5​ and z6​:

z5​ = 0.1 + (−0.1) ⋅ a3 ​+ 0.2 ⋅ a4

z5 = 0.17

z6​=0.1+0.1⋅a3​+(−0.2)⋅a4​

z6 = 0.03

On applying the rectifier linear activation function to z5z5​ and z6z6​:

a5 = max (0, 0.17) = 0.17

a6 = max (0, 0.03) = 0.03

So, the final output generated by the network is:

a5 = 0.17

a6=0.03

These are the values for Neurons 5 and 6, respectively, in response to the given input.

**Part b**. Calculate the **sum of squared errors** for this network on this input.

Solution:

To calculate the sum of squared errors (SSE), we'll compare the actual outputs of the network with the desired outputs and square the differences. The SSE is given by the formula:

SSE=n∑i=1ε2i

= ((0.7 – 0.17)2 + (0.4 – 0.03)2)

= 0.41779

Mean Squared error = 1/n SSE

= ½ (0.41799)

= 0.2088

**Part C.**

The error delta (δ) for each processing neuron in the network can be calculated using the following formulas:

For output neurons (5 and 6): δi=(yi−ai)⋅f′(zi)δi​=(yi​−ai​)⋅f′(zi​)

For hidden neurons (3 and 4): δi=(∑jwji⋅δj)⋅f′(zi)δi​=(∑j​wji​⋅δj​)⋅f′(zi​)

Therefore,

Output Neuron 6 (δ6δ6​):

δ6 = (0.4−0.03) × 1 = 0.37

Output Neuron 5 (δ5δ5​):

δ5 = (0.7−0.162) × 1 = 0.53

Hidden Neuron 4 (δ4δ4​):

δ4 = ((−0.2)×0.362+(−0.2)×0.538)×1 = 0.032

Hidden Neuron 3 (δ3δ3​):

δ3 = ((0.1)×0.362+(−0.1)×0.538)×0 = 0

These are the numerical values for the error deltas for each processing neuron in the network.

**Part D.**

The sensitivity of the error of the network to changes in each of the weights can be calculated using the following formula:

∂E∂wij=δj⋅ai∂wij​∂E​=δj​⋅ai​

where ∂E∂wij represents the sensitivity of the error EE with respect to the weight wij​, δj​ is the error delta at neuron j, and ai​ is the activated output at neuron ii.

Let's calculate the sensitivity of the error to each weight:

Sensitivity of the error to weights of Output Neuron 6:

∂E/∂w6,4 = δ6⋅a4 = 0.37⋅0.34 = 0.1258

∂E/∂w6,3 = δ6⋅a3 = 0.362⋅0 = 0

∂E/∂w6,0 = δ6⋅1 = 0.362

Sensitivity of the error to weights of Output Neuron 5:

∂E/δ5∂w5,4 = δ5⋅a4 = 0.538⋅0.34 = 0.18292

∂E/δ5∂w5,3 = δ5⋅a3 = 0.538⋅0 = 0

∂E/δ5∂w5,0 = δ5⋅1 = 0.538

Sensitivity of the error to weights of Hidden Neuron 4:

∂E/∂w4,2 = δ4⋅a2 = −0.180⋅0.6 = −0.108

∂E/∂w4,1 = δ4⋅a1 = −0.180⋅0.3 = −0.054

∂E/∂w4,0 = δ4⋅1 = −0.180

Sensitivity of the error to weights of Hidden Neuron 3:

∂E/∂w3,2 = δ3⋅a2 = 0⋅0.6 = 0

∂E/∂w3,1 = δ3⋅a1 = 0⋅0.3 = 0

∂E/∂w3,0 = δ3⋅1 = 0

These are the sensitivities of the error of the network to changes in each of the weights.

e. Assuming **a learning rate (**𝜶**) of 0.1**, update the **weights** of the network (𝑤6,4,𝑤6,3,𝑤6,0,𝑤5,4,𝑤5,3,𝑤5,0,𝑤4,2,𝑤4,1,𝑤4,0,𝑤3,2,𝑤3,1,𝑤3,0) after processing this input data.

Solution:

We'll calculate the updated weights using the following formula for each weight:

wij←wij+α⋅δj⋅aiwij​←wij​+α⋅δj​⋅ai​

Here are the updated weights:

Weight w6,4w6,4​:

w6,4 = -0.2+0.1⋅0.362⋅0.34 = -0.18742

Weight w6,3w6,3​:

w6,3 = 0.1+0.1⋅0.362⋅0=0.1

Weight w6,0w6,0​:

w6,0 = 0.1+0.1⋅0.362 = 0.137

Weight w5,4w5,4​:

w5,4 = 0.2+0.1⋅0.538⋅0.34 = 0.218292

Weight w5,3w5,3​:

w5,3 = -0.1+0.1⋅0.538⋅0 = -0.1

Weight w5,0w5,0​:

w5,0 = 0.1+0.1⋅0.538=0.1538

Weight w4,2w4,2​:

w4,2 = −0.2+0.1⋅−0.180⋅0.6 = 0.2892

Weight w4,1w4,1​:

w4,1 = 0.2 + 0.1⋅−0.180⋅0.3 = 0.194

Weight w4,0w4,0​:

w4,0 = 0.1+0.1⋅−0.180 = 0.082

Weight w3,2w3,2​:

w3,2 = −0.2+0.1⋅0⋅0.6 = -0.2

Weight w3,1w3,1​:

w3,1 = −0.1+0.1⋅0⋅0.3 = −0.1

Weight w3,0w3,0​:

w3,0 = 0.1+0.1⋅0 = 0.1

These are the updated weights after processing this input data with a learning rate of 0.1.

Part (f) Calculate **the new output** (𝒛 **and** 𝒂 **values for each neuron**) of this network on the input with the updated weights.

Solution:

Hidden Neuron 3:

z3=0.1+(−0.1)⋅0.3+(−0.2)⋅0.6=0.1−0.03−0.12=−0.05

a3=max(0,−0.05)= 0

Hidden Neuron 4:

z4=0.082+0.1946⋅0.3+0.2892⋅0.6=0.082+0.05838+0.17352=0.3139

a4=max(0,0.3139)=0.3139

Output Neuron 5:

z5=0.1538+(−0.1)⋅0+0.218292⋅0.3139=0.1538+0+0.068715858=0.222515858

a5=max(0,0.222515858)=0.22

Output Neuron 6:

z6=0.137+0.1⋅0+(−0.18742)⋅0.3139=0.137−0.05871=0.07

a6=max(0,0.07829)=0.07

These are the updated values of z and a for the four neurons.

(g) Calculate the **sum of squared errors** of this network on the input with the new weights.

Solution:

To calculate the sum of squared errors (SSE), we'll compare the actual outputs of the network with the desired outputs and square the differences. The SSE is given by the formula:

SSE=n∑i=1ε2i

= ((0.7 – 0.22)2 + (0.4 – 0.07)2)

= 0.33

MSE(mean squared error) = 1/n \* (SSE)

= ½ \* 0.33

= 0.165

(h) Show the **reduction in error** achieved by the network with new weights, compared to using the original weights.

Solution:

SSE of with the original weights: 0.41779

SSE of with the updated weights: 0.33

Therefore, the reduction in error: old – new

= 0.41779 – 0.33

= 0.08779

MSE of with the original weights: 0.2088

MSE of with the updated weights: 0.165

Therefore, the reduction in error: old – new

= 0.2088 – 0.165

= 0.0438