**Assignment 3**

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Part 1

1. A) Calculate the output generated by the network in response to this input. Show the process of output generation, including the z and 𝒂 values for each neuron in the network.

Solution:

For Neuron 3 (first hidden neuron):

z3 = (−0.1⋅0.3)+(−0.2⋅0.6) + 0.1 \* 1

= −0.03 −0.12 + 0.1

z3 = −0.05

For Neuron 4 (second hidden neuron):

z4 = (0.2⋅0.3)+(0.3⋅0.6)+0.1

= 0.06 + 0.18 + 0.1

Z4 = 0.34

On applying the rectifier linear activation function to z3​ and z4​:

a3 = max (0, −0.05) = 0

a4 = max (0, 0.16) = 0.34

Calculate z5​ and z6​:

z5​ = 0.1 + (−0.1) ⋅ a3 ​+ 0.2 ⋅ a4

z5 = 0.17

z6​=0.1+0.1⋅a3​+(−0.2)⋅a4​

z6 = 0.03

On applying the rectifier linear activation function to z5z5​ and z6z6​:

a5 = max (0, 0.17) = 0.17

a6 = max (0, 0.03) = 0.03

So, the final output generated by the network is:

a5 = 0.17

a6=0.03

These are the values for Neurons 5 and 6, respectively, in response to the given input.

b. Calculate the **sum of squared errors** for this network on this input.

Solution:

To calculate the sum of squared errors (SSE), we'll compare the actual outputs of the network with the desired outputs and square the differences. The SSE is given by the formula:

SSE=n∑i=1ε2i

= ((0.7 – 0.17)2 + (0.4 – 0.03)2)

= 0.41779

Part C.

The error delta (δδ) for each processing neuron in the network can be calculated using the following formulas:

For output neurons (5 and 6): δi=(yi−ai)⋅f′(zi)δi​=(yi​−ai​)⋅f′(zi​)

For hidden neurons (3 and 4): δi=(∑jwji⋅δj)⋅f′(zi)δi​=(∑j​wji​⋅δj​)⋅f′(zi​)

Therefore,

Output Neuron 6 (δ6δ6​):

δ6=(0.4−0.03)×1 = 0.37

Output Neuron 5 (δ5δ5​):

δ5=(0.7−0.162)×1 = 0.53

Hidden Neuron 4 (δ4δ4​):

δ4=((−0.2)×0.362+(−0.2)×0.538)×1 = 0.032

Hidden Neuron 3 (δ3δ3​):

δ3=((0.1)×0.362+(−0.1)×0.538)×0 = 0

These are the numerical values for the error deltas for each processing neuron in the network.

d.

The sensitivity of the error of the network to changes in each of the weights can be calculated using the following formula:

∂E∂wij=δj⋅ai∂wij​∂E​=δj​⋅ai​

where ∂E∂wij∂wij​∂E​ represents the sensitivity of the error EE with respect to the weight wijwij​, δjδj​ is the error delta at neuron jj, and aiai​ is the activated output at neuron ii.

Let's calculate the sensitivity of the error to each weight:

Sensitivity of the error to weights of Output Neuron 6:

∂E/∂w6,4 = δ6⋅a4 = 0.37⋅0.34 = 0.1258

∂E/∂w6,3 = δ6⋅a3 = 0.362⋅0 = 0

∂E/∂w6,0 = δ6⋅1 = 0.362

Sensitivity of the error to weights of Output Neuron 5:

∂E/δ5∂w5,4 = δ5⋅a4 = 0.538⋅0.34 = 0.18292

∂E/δ5∂w5,3 = δ5⋅a3 = 0.538⋅0 = 0

∂E/δ5∂w5,0 = δ5⋅1 = 0.538

Sensitivity of the error to weights of Hidden Neuron 4:

∂E/∂w4,2 = δ4⋅a2 = −0.180⋅0.6 = −0.108

∂E/∂w4,1 = δ4⋅a1 = −0.180⋅0.3 = −0.054

∂E/∂w4,0 = δ4⋅1 = −0.180

Sensitivity of the error to weights of Hidden Neuron 3:

∂E/∂w3,2 = δ3⋅a2 = 0⋅0.6 = 0

∂E/∂w3,1 = δ3⋅a1 = 0⋅0.3 = 0

∂E/∂w3,0 = δ3⋅1 = 0

These are the sensitivities of the error of the network to changes in each of the weights.

e. Assuming **a learning rate (**𝜶**) of 0.1**, update the **weights** of the network (𝑤6,4,𝑤6,3,𝑤6,0,𝑤5,4,𝑤5,3,𝑤5,0,𝑤4,2,𝑤4,1,𝑤4,0,𝑤3,2,𝑤3,1,𝑤3,0) after processing this input data.

Solution:

We'll calculate the updated weights using the following formula for each weight:

wij←wij+α⋅δj⋅aiwij​←wij​+α⋅δj​⋅ai​

Here are the updated weights:

Weight w6,4w6,4​:

w6,4 = -0.2+0.1⋅0.362⋅0.34 = -0.18742

Weight w6,3w6,3​:

w6,3 = 0.1+0.1⋅0.362⋅0=0.1

Weight w6,0w6,0​:

w6,0 = 0.1+0.1⋅0.362 = 0.137

Weight w5,4w5,4​:

w5,4 = 0.2+0.1⋅0.538⋅0.34 = 0.218292

Weight w5,3w5,3​:

w5,3 = -0.1+0.1⋅0.538⋅0 = -0.1

Weight w5,0w5,0​:

w5,0 = 0.1+0.1⋅0.538=0.1538

Weight w4,2w4,2​:

w4,2 = −0.2+0.1⋅−0.180⋅0.6 = 0.2892

Weight w4,1w4,1​:

w4,1 = 0.2 + 0.1⋅−0.180⋅0.3 = 0.194

Weight w4,0w4,0​:

w4,0 = 0.1+0.1⋅−0.180 = 0.082

Weight w3,2w3,2​:

w3,2 = −0.2+0.1⋅0⋅0.6 = -0.2

Weight w3,1w3,1​:

w3,1 = −0.1+0.1⋅0⋅0.3 = −0.1

Weight w3,0w3,0​:

w3,0 = 0.1+0.1⋅0 = 0.1

These are the updated weights after processing this input data with a learning rate of 0.1.

(f) Calculate **the new output** (𝒛 **and** 𝒂 **values for each neuron**) of this network on the input with the updated weights.

Solution:

Hidden Neuron 3:

z3=0.1+(−0.1)⋅0.3+(−0.2)⋅0.6=0.1−0.03−0.12=−0.05

a3=max(0,−0.05)= 0

Hidden Neuron 4:

z4=0.082+0.1946⋅0.3+0.2892⋅0.6=0.082+0.05838+0.17352=0.3139

a4=max(0,0.3139)=0.3139

Output Neuron 5:

z5=0.1538+(−0.1)⋅0+0.218292⋅0.3139=0.1538+0+0.068715858=0.222515858

a5=max(0,0.222515858)=0.22

Output Neuron 6:

z6=0.137+0.1⋅0+(−0.18742)⋅0.3139=0.137−0.05871=0.07

a6=max(0,0.07829)=0.07

These are the updated values of z and a for the four neurons.

(g) Calculate the **sum of squared errors** of this network on the input with the new weights.

Solution:

To calculate the sum of squared errors (SSE), we'll compare the actual outputs of the network with the desired outputs and square the differences. The SSE is given by the formula:

SSE=n∑i=1ε2i

= ((0.7 – 0.22)2 + (0.4 – 0.07)2)

= 0.33

(h) Show the **reduction in error** achieved by the network with new weights, compared to using the original weights.

Solution:

SSE of with the original weights: 0.41779

SSE of with the updated weights: 0.33

Therefore, the reduction in error: old – new

= 0.41779 – 0.33

= 0.08779